Simulation of Axon Activation by Electric Stimulation with Finite Difference Time Domain Method

Charles T.M. Choi and Shu-Hai Sun
Department of Computer Science and Institute of Biomedical Engineering, National Chiao Tung University
1001, Ta Hsueh Road, Hsinchu, 30010 Taiwan
E-mail Address: c.t.choi@ieee.org

Abstract —A cable model equivalent of an axon was placed in a homogeneous medium with an electrode was used to introduce excitations to stimulate the cable model in order to study axon activation by electric stimulation. Next, the transmembrane potentials and the ionic currents in the cable model were observed in the temporal domain. In this paper, the finite difference time domain (FDTD) method is being coupled with Hodgkin-Huxley models. A new scheme is being developed to predict axon activations. By observing the convergence of the transmembrane potential and axon activation, the simulation results show that the novel scheme is useful tool to simulate and study axon activation.

I. INTRODUCTION

The equivalent electric circuit representing the membrane of a squid giant axon was presented by Hodgkin and Huxley [1]. The Hodgkin-Huxley (HH) model is useful to simulate the membrane activation when a current is injected into a squid giant axon. Although the classical Hodgkin-Huxley experiments were carried out on the giant squid axon, differences in external potentials caused by the thickness of the axon are neglected in the following considerations. In 1976, McNeal first modeled an axon using many small axon segments described by the equivalent model of a membrane and then connecting them together with the resistances of the axoplasm [2]. This kind of cable model is useful to simulate and predict the axon response to extracellular electric stimulation [3].

In the current method, we simplified the medium surrounding the cable model to be homogeneous tissue. are often inhomogeneous. Here we introduce the finite difference time domain (FDTD) method [4]-[6] to simulate axon activation. In the FDTD method, the temporal and spatial domains are discretized, and the wave propagation can be simulated by computing the discrete electromagnetic field in the computational domain. By coupling FDTD with the HH model, this new method is useful to simulate axon activation.

II. METHOD

In this section, the cable model was introduced first, and the algorithm which couples the FDTD method with the HH model is described letter.

A. Cable Model

Hodgkin and Huxley gave a general description of the time course of the current which flows through the membrane of a squid giant axon when the potential difference across the membrane is suddenly changed from its steady state [1]. The results in [1], suggest that the behavior of a membrane may be represented by the electrical circuit shown in Fig. 1(a). Current can be carried through the membrane either by charging the membrane capacitance or by movement of ions through the nonlinear conductance in parallel with the membrane capacitance. The equation that describes the HH model is:

$$\begin{aligned} I &= I_{C_m} + I_K + I_{Na} + I_L \\ &= C_m \frac{dV_m}{dt} + g_K n(V_m - E_K) + g_{Na} m^3 h(V_m - E_{Na}) \\ &\quad + g_L(V_m - E_L) \end{aligned} \qquad (1)$$

where I is the total ionic current across the membrane due to flux of ions, C_m is the membrane capacity per unit area, g_K and g_{Na} are conductance of potassium and sodium channels, g_L is the leakage conductance of chloride or other, and V_m is the potential across the membrane. The parameters n, m, and h in (1) are the probability parameters determining the percentage of open channels, and all variables are change between 0 and 1 as functions of time and voltage. More details about the HH model were described in [1].

As shown in Fig. 1(b), the cable model based on the HH model was represented by the electric network. Applying Kirchhoff's current law to the electric network, a set of differential equations discrete in space and continuous in time, known as the cable equation, is useful to predict axon activation [2]-[3].

B. FDTD algorithm

For the FDTD method, Maxwell's curl equations are discretized in temporal and spatial domains, and the electric- and magnetic-field components are allocated on Yee cells filled in the computational domain. It is useful to analysis the distribution and the propagation of the electromagnetic field in the computational domain.

The FDTD method was extended to solve the distributed electromagnetic system with lumped elements and voltage and current sources [7]. Here the FDTD method is coupled with the equivalent electric current of the membrane, the HH model, and each HH model is connected by conductive components to perform a cable model. This method is useful to simulate the activation of a membrane and the propagation of a spike in an axon.

III. SIMULATION RESULT

By coupling FDTD method with HH model, the first test is the convergence of the membrane potential without any excitation. Under the steady state conditions, the

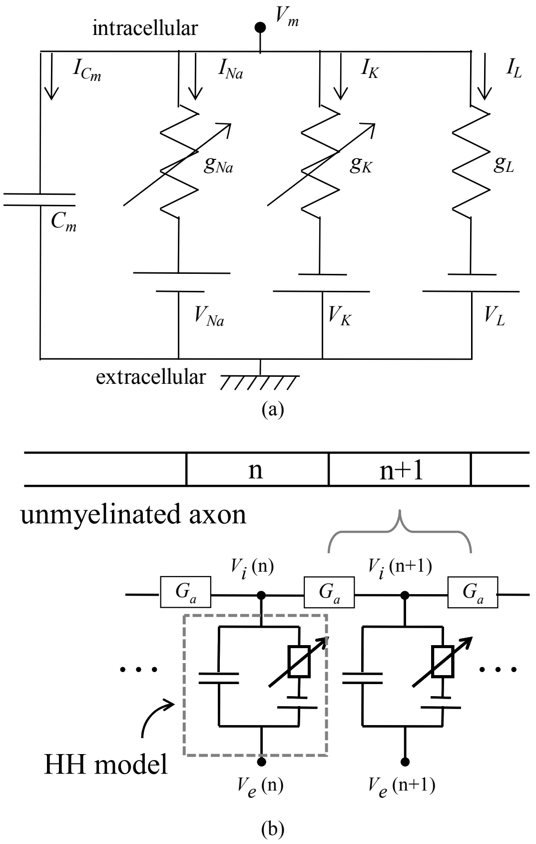

Fig. 1. Equivalent electric currents of (a) a membrane and (b) an unmyelinated axon

voltage across the membrane is equal to the voltage of the resting potential V_r, where the voltage of the resting potential can be analytically computed as follows:

$$V_r = \frac{g_K n^4 E_K + g_{Na} m^3 h E_{Na} + g_L E_L}{g_K n^4 + g_{Na} m^3 h + g_L} = 60.27 \text{ mV} \qquad (2)$$

Because the entire system is linear, the cell size in the FDTD framework can be set large to reduce the computational time. Here the cell size is set as 100 m, and two initial conditions of the membrane are respectively considered: all zeros and steady state. The values of the parameters setting in the initial conditions are listed in Table I, and the simulation results are shown in Fig. 2. From the simulation results, we can find the membrane potential converges to the resting potential when the initial conditions of the membrane are all zeros, and the membrane potential keeps at the resting potential when the initial conditions of the membrane are set as steady-state values. Therefore, by coupling FDTD method with HH model, this new method keeps the characteristic of the HH model, and it will be useful tool to study axon activation.

TABLE I
INITIAL CONDITIONS OF THE MEMBRANE IN THE CONVERGENCE TEST

initial condition	V_m	n	m	h
all zeros	0	0	0	0
steady state	60.27	0.3177	0.0529	0.5961

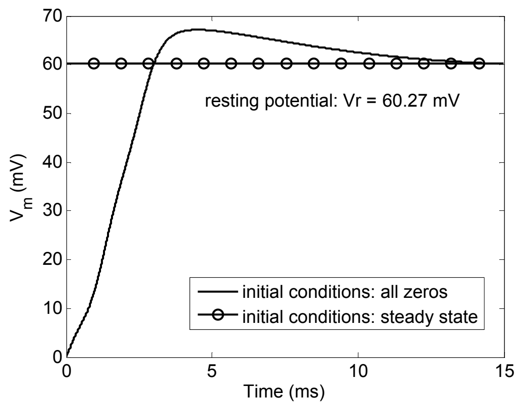

Fig. 2. Convergence of the membrane potential

IV. CONCLUSION

By coupling the FDTD method with the HH model, a new scheme is presented to simulate the activation of a membrane and the propagation of a spike in an axon. The simulation results show this method keeps the characteristic of the HH model, and it will be a useful tool to simulate axon activation and the propagation of a spike in an axon.

V. REFERENCES

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